BT-MRE-I&II(S)-04.21.0281 Reg. No. | | | | | | | | **B**

B.Tech. Degree I & II Semester Supplementary Examination in Marine Engineering, April 2021

MRE 1101 ENGINEERING MATHEMATICS 1

Time: 3 Hours

Maximum Marks: 100

I. (a) State Rolle's theorem and verify it for the function $(x-a)^m (x-b)^n$ where m and n are positive integers in $[a,b]$. (6)

(b) Evaluate $\lim_{x\to 0}\left\{\dfrac{xe^x - \log(1+x)}{x^2}\right\}$. (6)

(c) Find the asymptotes of the curve $(x+y)^2(x+y+2) = x+9y-2$. (8)

OR

II. (a) Evaluate $\lim_{x\to \frac{\pi}{2}}\left\{(\sin x)^{\tan x}\right\}$ (6)

(b) Find the n^{th} derivative of $e^x(2x+3)^3$. (6)

(c) If $y = e^{a\sin^{-1}x}$, prove that (8)

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$$

III. (a) If $U = \log\left(\dfrac{x^3+y^5}{x-y}\right)$, prove using Euler's theorem that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 4$. (6)

(b) If $u = \dfrac{xy}{x^2+y^2}, x = \cos ht, y = \sin ht$, find $\dfrac{du}{dt}$ using chain rule. (6)

(c) If $u = x^2\tan^{-1}\left(\dfrac{y}{x}\right) - y^2\tan^{-1}\left(\dfrac{x}{y}\right)$, show that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{x^2-y^2}{x^2+y^2}$. (8)

OR

IV. (a) Discuss the maxima and minima of $f(x,y) = x^3y^2(1-x-y)$. (10)

(b) If the sides of a plane triangle ABC vary in such a way that its circum radius remains constant, prove that $\dfrac{da}{\cos A} + \dfrac{db}{\cos B} + \dfrac{dc}{\cos C} = 0$. (10)

V. (a) Show that the tangents at the extremities of a focal chord intersect at right angles on the direatrix. (6)

(b) Find the condition that the straight line $lx + my + n = 0$ may be a normal to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. (6)

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(c) Find the centre, eccentricity, foci and length of the latus rectum of the hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$. (8)

OR

VI. (a) Prove that the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other in the curve $y^2 = a(x - 3a)$. (10)

(b) Find the equation of the hyperbola passing through (2,3) and has the straight lines $4x + 3y - 7 = 0$ and $x - 2y - 1 = 0$ as asymptotes. (10)

VII. (a) Find the reduction formula for $\int \sin^n(x)dx$ and hence evaluate $\int_0^a \frac{x^7 dx}{\sqrt{(a^2 - x^2)}}$. (10)

(b) Find the area enclosed between one arch of the cycloid $x = a(\theta - \sin\theta); y = a(1 - \cos\theta)$; and its base. (10)

OR

VIII. (a) Change the order of integration and hence evaluate $I = \int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dxdy}{\sqrt{(y^4 - a^2x^2)}}$. (10)

(b) Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. (10)

IX. (a) Given $\overline{a} = 2i + 2j - k, \overline{b} = 6i - 3j + 2k$, find $\overline{a} \times \overline{b}$ and a unit vector perpendicular to both $\overline{a}$ and $\overline{b}$. Also determine the sine of the angle between $\overline{a}$ and $\overline{b}$. (10)

(b) If $(\overline{a}, \overline{b}, \overline{c})$ and $(\overline{a}, \overline{b}, \overline{c})$ are reciprocal triads of vectors, show that $\overline{a} \times \overline{a} + \overline{b} \times \overline{b} + \overline{c} \times \overline{c} = \overline{0}$. (10)

OR

X. (a) Find the values of the constants a, b, c for which the vector $\overline{f} = (x + y + az)i + (bx + 3y + z)j + (3x + cy + z)k$ is irrotational. (10)

(b) Prove that (10)

(i) grad $\left(\frac{1}{r}\right) = \frac{-\overline{r}}{r^3}$.

(ii) $\nabla r^n = nr^{n-2}\,\overline{r}$

Where $\overline{r} = xi + yj + zk$.

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